DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

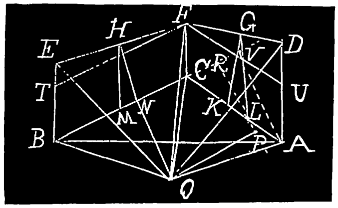
ARITHMETIC.

137. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

At the corners of a triangle sides a, b, c feet, are towers d, e, f feet high. At what point must a ladder be placed so that it will just reach to the top of each tower without moving? How long is the ladder? Substitute $a=200$, $b=180$, $c=150$, $d=60$, $e=50$, $f=30$; d, e, f at A, B, C, respectively.

Solution by the PROPOSER.

Let ABC be the triangle, $AD=d$, $BE=e$, $CF=f$, the towers. Join DF, EF, and draw UF parallel to AC, TF parallel to BC. From G, the mid-point of DF, draw GK parallel to AD, GL perpendicular to DF. From H, the mid-point of EF, draw HM parallel to BE, HN parallel to EF. Draw ON perpendicular to BC, and OL perpendicular to AC. Then O is the required foot of the ladder. For O is equally distant from D, E, F, since OL is perpendicular to the plane $ADFC$ at L, and ON is perpendicular to the plane $BCFE$ at N. Draw LR, AV perpendicular to BC, OP perpendicular to LR.



Then $DU=d-f$, $EF=e-f$, $GK=\frac{1}{2}(d+f)$, $HM=\frac{1}{2}(e+f)$.

In the similar triangles LGK and DFU, $LK:GK=DU:UF$.

$$\therefore\ LK=\frac{d^2-f^2}{2b},\quad CL=\tfrac{1}{2}b+\frac{d^2-f^2}{2b}=\frac{b^2+d^2-f^2}{2b}.$$

$$\text{Similarly } MN=\frac{e^2-f^2}{2a},\quad CN=\tfrac{1}{2}a-\frac{e^2-f^2}{2a}=\frac{a^2+f^2-e^2}{2a}.$$

$$AV=\frac{2\triangle}{a},\ \text{where } \triangle=\text{area } ABC.\quad VC=\sqrt{\frac{b^2-4\triangle^2}{a^2}}.$$

$$\therefore\ VC=\frac{a^2+b^2-c^2}{2a}.\quad RC:LC=VC:AC.$$

$$\therefore\ RC=\frac{(b^2+d^2-f^2)(a^2+b^2-c^2)}{4ab^2}.$$

$$RN=OP=RC+NC=\frac{(b^2+d^2-f^2)(a^2+b^2-c^2)+2b^2(a^2+f^2-e^2)}{4ab^2}.$$

$$OL:OP=AC:AV.$$

$$\therefore\ OL=\frac{(b^2+d^2-f^2)(a^2+b^2-c^2)+2b^2(a^2+f^2-e^2)}{8\triangle b}.$$

$OF=\sqrt{(OL^2+CL^2+CF^2)}$.

When $a=200$, $b=180$, $c=150$, $d=60$, $e=50$, $f=30$,

$OL^2=60363.9509$, $CL^2=9506.25$, $CF^2=900$.

$\therefore\ OF=266.03$ feet.

Also solved by *J. SCHEFFER.*

136. Proposed by F. M. PRIEST, Mona House, St. Louis, Mo.

"A pound of gold may be drawn into a wire that would extend around the earth." What would be the diameter of such a wire if the specific gravity of gold is 19.36 and the distance is 24,900 miles?

Solution by J. M. ARNOLD, Crompton, R. I.; G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.; and J. SCHEFFER, A. M., Hagerstown, Md.

62.4 pounds=weight of 1 cubic foot of water. Then, the specific gravity of gold being 19.36, the weight of 1 cubic foot of gold is 19.36×62.4 pounds, or 1208.064 pounds.

Hence, in 1 pound of gold there are $\frac{1728}{1208.64}$ or 1.43039 cu inches nearly.

$\therefore\ \frac{1}{4}\pi d^2\times24900\times5280\times12=1.43039$ cubic inches.

$\therefore\ d=.000034$ inches, nearly.

Mr. Arnold remarks that to measure so small a quantity one would have to estimate 1-12 of one of the divisions of a Brown and Sharp's Micrometer Gage, which reads to the hundredth of a millimeter.

Also solved by *ELMER SCHUYLER.*

139. Proposed by F. P. MATZ. M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College Mechanicsburg, Pa.

The *ratio* of the interest to the true discount on a certain principal for a certain time at a certain rate per cent. per annum, is $m=21$ to $n=20$. What is the rate per cent.?

Solution by P. S. BERG, B. Sc., Principal of Schools, Larimore, N. D.; and ELMER SCHUYLER, M. Sc., Professor of Mathematics, Boys' High School, Reading, Pa.

Let P be the principal ; r, the rate ; and t, the time in years.

Then the interest, I, is trP.

The true discount$=\frac{trP}{1+rt}$. $\therefore\ trP:\frac{trP}{1+tr}=m:n=21:20$.

$\therefore\ 1+rt=m/n=\frac{21}{20}$, and $rt=\frac{m-n}{n}=\frac{1}{20}$, or $r=\frac{1}{20t}$.

Thus r depends on the time.

If $t=1$ year, $r=5\%$.

Also solved by *G. B. M. ZERR, J. M. ARNOLD.* and *J. SCHEFFER.*